

THE RUBAKOV-CALLAN EFFECT AND BLACK HOLES ¹**T. KUBOTA***Department of Physics, Osaka University
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The Rubakov-Callan effect is reexamined by considering the gravitational effects caused by the heavy monopole mass. Assuming that the Higgs vacuum expectation value is as large as (or larger than) the Planck mass, we show that the calculational scheme of Rubakov and Callan may be extended in the presence of curved background field. It is argued that the density of the fermion condensate around a magnetically charged black hole is modified in an intricate way.

1 Introduction

It was early in spring of 1983 when I moved to Osaka University that I began to have the good fortune to work with Professor Keiji Kikkawa as one of his research associates. No sooner than I arrived at Osaka, Kikkawa suggested to me to work with him on the finite temperature effect on the monopole catalysis of the baryon number violation, then a brand-new phenomenon predicted by Rubakov [1] and Callan [2]. We started to work on this fresh idea together with H.S. Song and the paper [3] is an outcome of our joint efforts.

The Rubakov-Callan effect attracted a lot of broad attention because of its phenomenological and astrophysical importance. In my opinion, however, it is even more attractive because of the novelty and boldness shown at the cutting edge of the development of non-perturbative field theories. Here I resume the Rubakov-Callan effect, but as a theoretical laboratory with a hope of gaining an insight into gravitational effects in quantum field theories.

Rubakov and Callan have shown that the fermion condensate around a magnetic monopole is long-ranged, i.e., falling off by an inverse power of the distance from the

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monopole. Kikkawa et al. [3] on the other hand have claimed that the condensate at finite temperature becomes short-ranged, and that the typical length scale is set by the inverse of temperature. If we consider the gravitational effect caused by the heavy monopole mass, we have another length scale, namely, the Planck length. It looks more than likely that the spatial dependence of the fermion condensate should be at variance with Rubakov-Callan's at this length scale. I will argue in this short note that this is in fact the case, and will give necessary calculational formulae.

2 Magnetically Charged Black Holes

To make things as clear as possible, we consider the conventional Einstein gravity coupled with $SU(2)$ gauge theory which spontaneously breaks down to $U(1)$ group by an $SU(2)$ -triplet Higgs scalar Φ^a . We are concerned with the behavior of left-handed fermion Ψ around a magnetic monopole in the presence of the gravitational background. The action consists of three parts

$$S = -\frac{1}{16\pi G} \int d^4x \sqrt{-g} R + S_{\text{boson}} + S_{\text{fermion}}, \quad (1)$$

where the bosonic and fermionic actions are given, respectively, by

$$S_{\text{boson}} = \int d^4x \sqrt{-g} \left\{ -\frac{1}{4} (F_{\mu\nu}^a)^2 + \frac{1}{2} |D_\mu \Phi^a|^2 - \frac{\lambda}{2} (|\Phi^a|^2 - v^2)^2 \right\}, \quad (2)$$

$$S_{\text{fermion}} = i \int d^4x \sqrt{-g} \bar{\Psi} e_\ell^\mu \gamma^\ell \left(\partial_\mu - \frac{i}{4} \omega_\mu^{mn} \sigma_{mn} - \frac{i}{2} e A_\mu^a \tau^a \right) \frac{1 - \gamma_5}{2} \Psi. \quad (3)$$

Here the gauge coupling is denoted by e . We will consider the static and spherically symmetric metric

$$ds^2 = B(r) dt^2 - A(r) dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (4)$$

as a solution to the Einstein equation and e_ℓ^μ and ω_μ^{mn} in Eq. (3) are the vierbein and spin connections respectively associated with the above metric.

The 'tHooft-Polyakov monopole in the gravitational background (4) has been considered in literatures [4,5,6]. Numerical solutions to the Einstein-Yang-Mills-Higgs equations are also available. There are two mass scales in this system: one is the Higgs vacuum expectation value v , and the other is the Planck mass M_P . If v is as large as M_P and the Schwarzschild radius is comparable to the monopole size, then the monopole should become a black hole. Lee, Nair and Weinberg [6] classified the classical solutions, drawing a "phase diagram" according to the value of v and the monopole mass. They also discussed the relation of their solutions to the Reissner-Nordström solution.

We are now interested in solving the monopole-fermion dynamics, considering (strong) quantum fluctuations of Yang-Mills field in the presence of the gravitational

classical background (4). Following Rubakov and Callan, we will consider a particular type of quantum fluctuations around the 'tHooft-Polyakov monopole configuration,

$$\Phi^a \tau^a = v h(r) \tau^1, \quad (5)$$

$$\begin{aligned} A_\mu^a \tau^a e_0^\mu &= \frac{2}{e} a_0(r, t) \tau^1, & A_\mu^a \tau^a e_1^\mu &= \frac{2}{e} a_1(r, t) \tau^1, \\ A_\mu^a \tau^a e_2^\mu &= \frac{u(r)}{er} \tau^2, & A_\mu^a \tau^a e_3^\mu &= \frac{u(r)}{er} \tau^3 + \frac{\cot \theta}{er} \tau^1. \end{aligned} \quad (6)$$

Here $a_0(r, t)$ and $a_1(r, t)$ are the quantum fluctuations and $u(r)$ and $h(r)$ are the classical part.

To put our system into a solvable form, we have to restrict ourselves to a limiting case, in which the radius of the monopole is vanishingly small. More specifically we will assume that v is much larger than M_P . To take this limiting case implies $h(r) = 1$ and $u(r) = 0$ outside the horizon. The Higgs and gauge fields take the same configurations as those at large distance even at the horizon. The bosonic action (2) then becomes

$$S_{\text{boson}} = \frac{8\pi}{e^2} \int dt dr \sqrt{A(r)B(r)} r^2 \left\{ \frac{1}{\sqrt{B(r)}} \frac{\partial a_1(r, t)}{\partial t} - \frac{1}{\sqrt{A(r)}} \frac{\partial a_0(r, t)}{\partial r} \right\}^2 \quad (7)$$

up to an additive constant. This is an action analogous to the two-dimensional QED in the gravitational background and $a_0(r, t)$ and $a_1(r, t)$ are the vector potential.

3 The Fermion Dynamics

A fermion moving around the monopole comes close to the core of the monopole if it is in S -wave. From here on we will consider only the S -wave fermions which is defined, if we use the chiral basis for the gamma matrices, by

$$\sqrt{r^2 \sin \theta \sqrt{B(r)}} \Psi^T = (0, \chi), \quad \chi_{\alpha\ell}(r, t) = \delta_{\alpha\ell} \chi_1(r, t) - i(\tau^1)_{\alpha\ell} \chi_2(r, t). \quad (8)$$

Here α and ℓ are spin and isospin indices, respectively. The S -wave part of the fermionic action is reminiscent of the two-dimensional fermion with two components if we set $\chi^T = (\chi_1, \chi_2)$,

$$S_{\text{fermion}} = -4\pi \int dt dr \sqrt{A(r)} \chi^\dagger D_{J=0} \chi, \quad (9)$$

where the Dirac operator in this case becomes

$$D_{J=0} = -i \left(\frac{1}{\sqrt{B(r)}} \frac{\partial}{\partial t} + i\tau^2 a_0(r, t) \right) + i\tau^2 \left(\frac{1}{\sqrt{A(r)}} \frac{\partial}{\partial r} - i\tau^2 a_1(r, t) \right). \quad (10)$$

The fermionic action (9) and the bosonic one (7) are saying that our dynamical system looks similar to the Schwinger model. There is, however, an important difference, that is, the boundary condition to be imposed on the fermion field. In the case of Rubakov and Callan, the boundary condition was imposed at $r = 0$, so that the approximation of vanishing monopole size is justified. In our case we have assumed $v \gg M_P$ and the Schwarzschild radius is greater than the radius of the monopole core. It is the most reasonable choice of the boundary condition that the flow of the probability should cease to go inside at the horizon (located at $r = r_H$) so that the hermiticity of the Hamiltonian is preserved. This is achieved by setting

$$(\tau^1 + i\tau^2)\chi(r_H, t) = 0. \quad (11)$$

Now we will take the gauge fixing condition $A_0^a = 0$ and will discard $a_0(r, t)$. The technique used by Rubakov and Callan to solve the monopole-fermion dynamics was the bosonization method, which is also vital in our case. Let us denote the bosonized field by $\varphi(r, t)$. The fermionic kinetic term is expressed by the bosonic one, and furthermore $\varphi(r, t)$ is related to the gauge field via

$$\square\varphi(r, t) = \frac{1}{\sqrt{B(r)}} \frac{\partial a_1(r, t)}{\partial t}, \quad (12)$$

where \square is the two dimensional Laplacian associated with $ds^2 = B(r)dt^2 - A(r)dr^2$.

A close look at Eqs. (7) and (12) shows that the kinetic term of the gauge field may be easily expressed by $\varphi(r, t)$ and we arrive at the effective action

$$S_{\text{eff}} = S_{\text{boson}} + S_{\text{fermion}} = \frac{1}{2} \int dt dr \sqrt{A(r)B(r)} \varphi(r, t) L_{rt} \varphi(r, t), \quad (13)$$

where

$$L_{rt} = \frac{8\pi}{e^2} \frac{1}{\sqrt{A(r)B(r)}} \square \sqrt{A(r)B(r)} r^2 \square + \frac{N_f}{\pi} \square. \quad (14)$$

The number of fermion flavors is denoted by N_f . Thus in the curved background as well, the two kinds of dynamical degrees of freedom due to fermionic and bosonic parts are described by a single scalar field $\varphi(r, t)$. The boundary condition (11) is transcribed into $\partial_r \varphi = 0$ at $r = r_H$.

4 Gravitational Effects on the Fermion Condensate

We are now in a position to see how the effect of the curved background appears in particular on the fermion condensate around a magnetic monopole. We will consider two flavor fermion doublets ($N_f = 2$). The fermion condensate that we are concerned with is $\langle f(r, t) \rangle$, where

$$f(r, t) = \chi_1^{(1)}(r, t) \chi_1^{(2)}(r, t) + \chi_2^{(1)}(r, t) \chi_2^{(2)}(r, t). \quad (15)$$

The superscript denotes the flavor index.

The dynamics described by (13) can be solved if the full Green's function $(L_{rt})^{-1}$ is known exactly. The operator (14) is very much simplified if we restrict our considerations to the Reissner-Nordström case, i.e., $A(r)B(r) = 1$. We will denote this Green's function by $\mathcal{P}(r, t; r', t')$ for this particular case and the boundary condition is $\partial_r \mathcal{P} = 0$ at the horizon $r = r_H$. According to the analysis by Rubakov and Callan, the density of the fermion condensate is given by $\langle f(r, t) \rangle \sim \exp\{2\mathcal{P}(r, t; r, t)\}$, the coincidence limit of this Green's function. It should be noticed that the Green's function $\mathcal{P}(r, t; r', t')$ is expressed as a combination of two other types of Green's functions defined by

$$\begin{aligned} \square \mathcal{D}(r, t; r', t') &= \delta(r - r')\delta(t - t'), \\ \left(\square + \frac{\kappa}{r^2}\right) \mathcal{R}_\kappa(r, t; r', t') &= \delta(r - r')\delta(t - t'). \end{aligned} \quad (16)$$

Here we used the notation $\kappa = N_f e^2 / 8\pi^2$.

It is rather straightforward to obtain $\mathcal{D}(r, t; r', t')$ by using the method of DeWitt and Schwinger [7]. Applying their method to our case, one finds that the Green's function is expressed by the Hankel function $H_0^{(2)}(\mu\sqrt{-2\sigma})$ and its derivatives. Here $\sigma = \sigma(r, t; r', t')$ is the two-dimensional geodesic interval between (r, t) and (r', t') and μ is an infrared cutoff. The leading term in the DeWitt-Schwinger expansion turns out to be

$$\frac{\Delta}{4\pi} \log[-2\mu^2 \sigma(r, t; r', t')], \quad (17)$$

where Δ is the bi-scalar function defined by DeWitt. The boundary condition is made satisfied by putting mirror images at $2r_H - r$ and $2r_H - r'$.

The coincidence limit of the Green's function $\mathcal{D}(r, t; r, t)$ contributes partly to $\mathcal{P}(r, t; r, t)$, and it entails the Coulomb interaction between (r, t) and its mirror image. The density of the fermion condensate is described thus by the bi-scalar functions Δ and σ .

As to the other Green's function $\mathcal{R}_\kappa(r, t; r', t')$, the calculational method of DeWitt [7] does not apply in its original form. It is, however, easy to see that the term κ/r^2 in (16) is playing the role of infrared regulator (or the position-dependent mass), and that there must exist a term like $\log r$ in the coincidence limit $\mathcal{R}_\kappa(r, t; r, t)$, contributing to the fermion condensate.

5 Summary

In the present paper we have seen that the mathematical framework of Rubakov and Callan may be generalized in an analogous way to the case of the curved background. The density of the fermion condensation is deformed not simply by the deformation of the space-time but also intricately by the mirror image. Throughout the present

note the gravity has been treated classically and quantum aspects of gravity are all neglected. It would be of particular interest if we could include strong gravitational fluctuations in this framework.

Gregory and Harvey [8] studied the zero-energy solution of the Dirac equation in the presence of the magnetic monopole and gravitational background. They argued the possibility of baryon number violating scattering processes off the magnetically charged black hole. The relation between their approach and the present one will be discussed elsewhere.

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References

- [1] V.A. Rubakov, Nucl. Phys. **B 203** (1982) 311.
- [2] C.G. Callan, Phys. Rev. **D 25** (1982) 2141; *ibidem* **D 26** (1982) 2058.
- [3] K. Kikkawa, T. Kubota and H.S. Song, Prog. Theor. Phys. **71** (1984) 1346.
- [4] F.A. Bais and R.J. Russel, Phys. Rev. **D 11** (1975) 2692; Y.M. Cho and P.G.O. Freund, Phys. Rev. **D 12** (1975) 1588.
- [5] P. van Nieuwenhuizen, D. Wilkinson and M.J. Perry, Phys. Rev. **D 13** (1976) 778.
- [6] K. Lee, V.P. Nair and E.J. Weinberg, Phys. Rev. **D 45** (1992) 2751; Phys. Rev. Lett. **68** (1992) 1100.
- [7] B.S. DeWitt, *Dynamical Theory of Groups and Fields* (Gordon and Breach, New York, 1965); J. Schwinger, Phys. Rev. **82** (1951) 664.
- [8] R. Gregory and J.A. Harvey, Phys. Rev. **D 46** (1992) 3302.

Note added:

After submitting the present paper for publication, I was informed of the related work by Piljin Yi [Phys. Rev. **D 49** (1994) 5295]. He also worked out the generalization of the Rubakov-Callan's bosonized effective action around general static spherically symmetric black holes. I would like to thank Dr. Piljin Yi for calling my attention to his paper.